

A Many-Worlds Product Paradigm for Quantum Inertia and Quantum Gravity

By William D. Eshleman

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Abstract. The conventional method for specification of a formalism for reality is to assume and determine for the chosen space, the following collection of mathematical objects: 1) state as a vector in a vector space, 2) observable as an operator that acts on the vector space, and 3) an algebra determined by the set of operators and certain constraints. Usually, this requires a detailed analysis of how vectors add in the chosen space. That is, the evolution of the state of a vector is a process whereby differential vectors are added to a vector. The original many-worlds interpretation took this approach and assumed the Hilbert space.

Detailed analysis of how vectors could magnify one another in the chosen space is usually avoided due to a change of the type of a vector upon multiplication; e.g., the product of two position tensors is not a tensor, but sums of tensors remain tensors. The report presented here attempts to investigate hypothetical mathematical products that predict equivalent results, as compared to conventional results, for relatively low-energy trajectories, and then to compare the differences for high-energy motion.

The assumed collection of mathematical objects for this report is: 1) state as a product of factors (resembling vectors) in a world of an observer, 2) observable as the logarithm of a particular factor that acts on the space of the observer, and 3) the algebra of convergent products, the notion of the equivalence of products, and the notion of the entropy of products. That is, the evolution of the state of a vector is a process whereby vectors magnify to produce change; the Product Paradigm.

Preface reprinted from *The Many-Worlds Interpretation of Quantum Mechanics* (A Fundamental Exposition by HUGH EVERETT, III, with papers by J. A. WHEELER, B. S. DEWITT, et. al. and edited by BRYCE S. DEWITT and NEILL GRAHAM, 1973)

** This is the beginning of Dr. DeWitt's preface of Dr. Everett's work **

PREFACE

In 1957, in his Princeton doctoral dissertation, Hugh Everett, III, proposed a new interpretation of quantum mechanics that denies the existence of a separate classical realm and asserts that it makes sense to talk about a state vector for the whole universe. This state vector never collapses and hence reality as a whole is rigorously deterministic. This reality, which is described jointly by the dynamical variables and the state vector, is not the reality we customarily think of, but is a reality composed of many worlds. By virtue of the temporal development of the dynamical variables the state vector decomposes naturally into orthogonal vectors, reflecting a continual splitting of the universe into a multitude of mutually unobservable but equally real worlds, in each of which every good measurement has yielded a definite result and in most of which the familiar statistical quantum laws hold.

In addition to his short thesis Everett wrote a much larger exposition of his ideas, which was never published. The present volume contains both of these works, together with a handful of papers by others on the same theme. Looked at in one way, Everett's interpretation calls for return to naive realism and the old fashioned idea that there can be direct correspondence between formalism and reality. Because physicists have become more sophisticated than this, and above all because the implications of his approach appear to them so bizarre, few have taken Everett seriously. Nevertheless his basic premise provides such a stimulating framework for discussions of the quantum theory of measurement that this volume should be on every quantum theoretician's shelf.

"... a picture, incomplete yet not false, of the universe as Ts'ui Pen conceived it to be. Differing from Newton and Schopenhauer, ... [he] did not think of time as absolute and uniform. He believed in an infinite series of times, in a dizzily growing, ever spreading network of diverging, converging and parallel times. This web of time – the strands of which approach one another, bifurcate, intersect or ignore each other through the centuries – embraces every possibility. We do not exist in most of them. In some you exist and not I, while in others I do, and you do not, and in yet others both of us exist. In this one, in which chance has favored me, you have come to my gate. In another, you, crossing the garden, have found me dead. In yet another, I say these very same words, but am an error, a phantom."

Jorge Luis Borges, *The Garden of Forking Paths*

“Actualities seem to float in a wider sea of possibilities from out of which they were chosen; and somewhere, indeterminism says, such possibilities exist, and form a part of the truth.”

William James

** This is the end of Dr. DeWitt's preface of Dr. Everett's work **

Einstein on Space

“When a smaller box s is situated, relatively at rest, inside the hollow space of a larger box S , then the hollow space of s is a part of the hollow space of S , and the same “space”, which contains both of them, belongs to each of the boxes. When s is in motion with respect to S , however, the concept is less simple. One is then inclined to think that s encloses always the same space, but a variable part of the space S . It then becomes necessary to apportion to each box its particular space, not thought of as bounded, and to assume that these two spaces are in motion with respect to each other.

Before one has become aware of this complication, space appears as an unbounded medium or container in which material objects swim around. But it must now be remembered that there is an infinite number of spaces, which are in motion with respect to each other. The concept of space as something existing objectively and independent of things belongs to pre-scientific thought, but not so the idea of the existence of an infinite number of spaces in motion relatively to each other. This latter idea is indeed logically unavoidable, but is far from having played a considerable role even in scientific thought.”

1 Introduction

We begin, as a way of entering our subject, by the development of an infinite product identity for the Lorentz (inertial) factor of Einstein’s Special Relativity. This identity was discovered by fortuitous accident, and what was initially a curious mathematical object, has resulted in the extension of infinite product theory and the suggestion that high-energy cosmology need not proceed toward chaos and mathematical breakdown. The definitions and proofs that follow this introduction are included to reinforce confidence that the interpretation is sound in a mathematical sense.

Since so much of the method of this theory has been taken directly from the work of Everett, it helps to be exposed to some interpretation of Everett’s perspective (as I see it); Everett’s view is probably more of a paradigm shift in causality. Whereas the Newtonian model requires that the deviation from straight-line motion in a gravitational field be due to forces “acting-at-a-distance” under the control of The Universal Law of Gravitation, the Everett perspective is probably closer to Einstein’s “warp in the continuum” without the need for space-time to be that continuum. Motion is accomplished on paths through symmetrical peaks and valleys of an entropy continuum. “Action-at-a-distance” does not apply because the entropy structure of space-time, containing mass and motion, does not possess the information required to distinguish between meters, seconds, or kilograms. In other words, motion does not depend on “invisible-springs”, “acting-at-a-distance”, connecting massive bodies over empty space, but depends instead on “invisible and symmetrical dents” in the entropy continuum that can constrain motion to within the “dents” like a ball-bearing in a coffee cup.

A consequence of this analysis is that a mathematical structure identical to the proposed inertial factor is the only candidate for the gravitational factor that preserves the

classical model for orbital motion into the realm of high-energy astrophysics. A statement of the equivalence of inertial and gravitational information of these two factors is this theory's General Relativity of Motion.

2 A Priori Notions of Relative State

Our most accepted notion of state is that it is due to the property of the exponential function that, for h a constant,

$$\frac{d\psi}{dt}(t) = h \psi(t) ,$$

has the solution, for $\psi_0 f_0^0 = \psi(0)$ and $\psi_0 f_1^t = \psi(t)$,

$$\psi_0 f_1^t = \psi_0 f_0^t (e^h)^t = \psi_0 f_0^t \left(1 + h + \frac{(h)^2}{2!} + \frac{(h)^3}{3!} + \dots\right)^t ,$$

or,

$$\frac{f_1}{f_0} = e^h = \left(1 + h + \frac{(h)^2}{2!} + \frac{(h)^3}{3!} + \dots\right) ,$$

the classical notion of subjective relative state independent of observation and time.

On the other hand, relativity theory predicts what observers perceive objectively; distorted by the process of observation itself. For example, inertially, before and after an acceleration,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

or,

$$m^2 = m_0^2 + \frac{v^2}{c^2} m^2$$

where m_0 is the rest mass and m is the mass in motion.

And, gravitationally (I suggest), before and after a displacement from infinity,

$$m = m_0 + \frac{GM}{Rc^2} m$$

or,

$$m = \frac{m_0}{1 - GM/R/c^2}$$

where M is a central mass separated from m by a distance of R and m_0 is the mass when infinitely distant from M.

Therefore, our relativistic/objective notion of relative state is:

$$\begin{aligned} \frac{f_1}{f_0} &= 1 + x + x^2 + x^3 + \dots, \\ &= \frac{1}{1 - x}, \text{ [Model 4]} \end{aligned}$$

where $x = v^2/c^2$, or (I suggest) $x = GM/R/c^2$, or (I suggest) $x = h$.

The goal is now to separate the subjective relative state from the objective relative state to reveal the physics as it truly proceeds. That is,

$$\text{Objective Distortion} = \frac{1}{(1 - x)e^x} = \frac{1}{e^x - xe^x},$$

a most difficult task, fraught with ambiguity and yielding only troublesome interpretations. Another method is to invent a “factorial operator” that divides every i^{th} term by $i!$. This operator is also hard to justify on physical grounds.

There does exist, though, a form for $1/(1 - x)$ that yields a separation of a candidate for subjective relative state, e.g.,

$$\frac{f_1}{f_0} = \prod_{i=0}^{\infty} (1 + x^{2^i})^{2^{-i}}, \text{ [Model 5]}$$

The above relationship is this theory’s assumed notion of subjective relative state. Figure 1. compares Model 5 (candidate for subjective relative state), Model 4 (relativistic objective relative state), e^x (classical subjective relative state), and $1 + x$ (simple relative state).

The reasoning concerning many-worlds is that without the effect of many-worlds, the relative state would follow the simple relationship $(1 + x)$. That is, the additional factors would be due to the presence of many-parallel-worlds. Conversely, if all of the additional factors beyond $(1 + x)$ are observational distortions, then simple relative state $(1 + x)$ is the only true candidate for subjective relative state. This undecidability and the incompatibility of classical and relativistic state are at the core of the difficulties in the unification of quantum mechanics and general relativity.

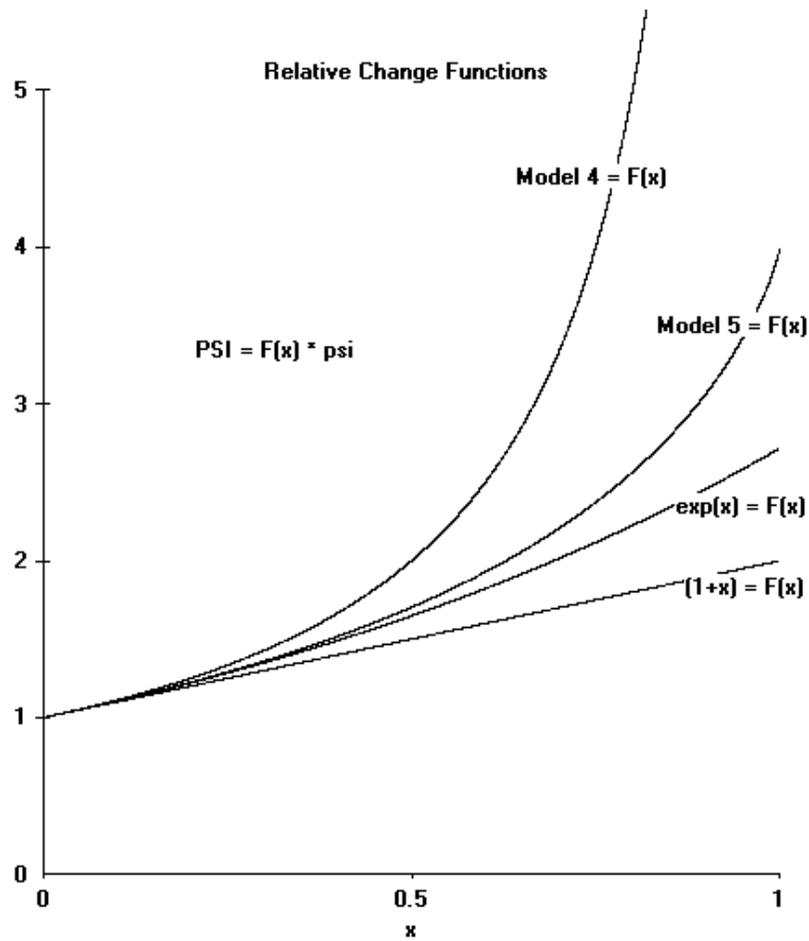


Figure 1. Comparison of Clocks

3 Proof of the Mathematical Object

I. Lemma.

$$1 - x^{2^N} = (1 - x) \prod_{n=0}^{N-1} (1 + x^{2^n}), \quad N = 1, 2, \dots$$

Proof

a) at $N = 1$,

$$1 - x^2 = (1 - x)(1 + x)$$

b) Inductively, the lemma is true for $N = 1, \dots, M$

$$\begin{aligned}
1 - x^{2^{M+1}} &= 1 - (x^{2^M})^2 \\
&= (1 - x^{2^M})(1 + x^{2^M}) \\
&= (1 - x) \prod_{n=0}^{M-1} (1 + x^{2^n})(1 + x^{2^M}) \\
&= (1 - x) \prod_{n=0}^M (1 + x^{2^n})
\end{aligned}$$

II. Lemma.

$$\frac{1}{1-x} = \prod_{n=0}^{\infty} (1 + x^{2^n}), \text{ for } 0 \leq x < 1$$

Proof

$$1 - x^{2^N} = (1 - x) \prod_{n=0}^{N-1} (1 + x^{2^n})$$

or,

$$1 = (1 - x) \prod_{n=0}^{\infty} (1 + x^{2^n})$$

III. Lemma.

$$\prod_{n=m}^{\infty} (1 + x^{2^n}) = \frac{1}{1 - x^{2^m}}$$

Proof

$$\begin{aligned} \prod_{n=m}^{\infty} (1 + x^{2^n}) &= \prod_{n=0}^{\infty} (1 + x^{2^n}) \left[\prod_{n=0}^{m-1} (1 + x^{2^n}) \right]^{-1} \\ &= \frac{1}{1 - x} \left[\frac{1 - x^{2^m}}{1 - x} \right]^{-1} \\ &= \frac{1}{1 - x^{2^m}} \end{aligned}$$

IV. Lemma.

$$\sum_{n=0}^m y^n = \frac{1 - y^{m+1}}{1 - y}$$

Proof

$$\begin{aligned} \sum_{n=0}^m y^n &= 1 + \sum_{n=1}^m y^n \\ &= 1 + y \sum_{n=0}^{m-1} y^n - y^{m+1} \\ (1 - y) \sum_{n=0}^m y^n &= 1 - y^{m+1} \\ \sum_{n=0}^m y^n &= \frac{1 - y^{m+1}}{1 - y} \end{aligned}$$

Corollary

$$1 - 2^{-m} = \left[\sum_{i=0}^{m-1} 2^i \right] 2^{-m}$$

V. Lemma.

$$\prod_{m=1}^{\infty} (1 + x^{2^m})^{1-2^{-m}} = \prod_{m=1}^{\infty} \prod_{n=m}^{\infty} (1 + x^{2^n})^{2^{-m}}$$

Proof

$$\begin{aligned} \prod_{m=1}^{\infty} (1 + x^{2^m})^{1-2^{-m}} &= \prod_{m=1}^{\infty} (1 + x^{2^m})^{(\sum_{i=0}^{m-1} 2^i) 2^{-m}} \\ &= \prod_{m=1}^{\infty} (1 + x^{2^m})^{(\sum_{i=0}^{m-1} 2^{i-m})} \\ &= \lim_{N \rightarrow \infty} \prod_{n=1}^N \prod_{i=1}^n (1 + x^{2^n})^{2^{-i}} \\ &= \lim_{N \rightarrow \infty} \prod_{\substack{1 < n \leq N \\ 1 \leq i \leq n}} (1 + x^{2^n})^{2^{-i}} \\ &= \lim_{N \rightarrow \infty} \prod_{\substack{i \leq n \leq N \\ 1 \leq i \leq N}} (1 + x^{2^n})^{2^{-i}} \\ &= \prod_{m=1}^{\infty} \prod_{n=m}^{\infty} (1 + x^{2^n})^{2^{-m}} \end{aligned}$$

VI. Theorem.

$$\frac{1}{1-x} = (1+x) \prod_{n=1}^{\infty} \left[\frac{(1+x^{2^n})}{(1-x^{2^n})} \right]^{2^{-n}}, \text{ for } 0 \leq x < 1$$

Proof

$$\begin{aligned}\frac{1}{1-x} &= \prod_{n=0}^{\infty} (1+x^{2^n}) \\ &= (1+x) \prod_{n=1}^{\infty} (1+x^{2^n})^{2^{-n}} (1+x^{2^n})^{1-2^{-n}} \\ &= [(1+x) \prod_{n=1}^{\infty} (1+x^{2^n})^{2^{-n}}] [\prod_{n=1}^{\infty} (1+x^{2^n})^{1-2^{-n}}] \\ &= [(1+x) \prod_{n=1}^{\infty} (1+x^{2^n})^{2^{-n}}] [\prod_{n=1}^{\infty} \prod_{i=n}^{\infty} (1+x^{2^i})^{2^{-n}}] \\ &= [(1+x) \prod_{n=1}^{\infty} (1+x^{2^n})^{2^{-n}}] [\prod_{n=1}^{\infty} (\frac{1}{1-x^{2^n}})^{2^{-n}}] \\ &= (1+x) \prod_{n=1}^{\infty} [\frac{(1+x^{2^n})}{(1-x^{2^n})}]^{2^{-n}}, \text{ for } 0 \leq x < 1\end{aligned}$$

4 Notion of Equivalence of Infinite Products

VII. Definition.

For $\alpha_n \geq 0$, $\beta_n \geq 0$ and $\rho \geq 0$,

$$\text{Infinite product } \mathbf{A} = \prod_{n=1}^{\infty} (1 + \alpha_n) \equiv$$

$$\text{Infinite product } \mathbf{B} = \prod_{n=1}^{\infty} (1 + \beta_n)$$

If, for value A of \mathbf{A} and value B of \mathbf{B} , $A = B^\rho$
and, if there is a permutation f , such that,

for, $a_n = (1 + \alpha_f)$,

and, $b_n = (1 + \beta_n)$,

the relation, $a_n = b_n^\rho$, holds for all n .

Remark. This is an equivalence relation among a class of sequences of the form $(1 + a_n)$ where a_n approaches 0 as n approaches infinity and for which the associated infinite product is finite.

VIII. Theorem. (Properties of equivalent infinite products) Infinite products may be confirmed equivalent without the need for the knowledge of the value of ρ . That is, equivalence depends on the existence of ρ , not its value.

Proof

For,

$$C = \sum_{n=1}^{\infty} \log(a_n),$$

and,

$$D = \sum_{n=1}^{\infty} \log(b_n),$$

1) If \mathbf{A} and \mathbf{B} are equivalent,

$$\frac{\log(a_n)}{C} = \frac{\log(b_n)}{D}$$

$$= \log_A(a_n)$$

$$= \log_B(b_n)$$

2) (The Normal Logarithm)

$$1 = \sum_{n=1}^{\infty} \log_A(a_n)$$

and,

$$1 = \sum_{n=1}^{\infty} \log_B(b_n)$$

Remark Equivalent infinite products need not be Equal Valued and Equal Valued infinite products need not be Equivalent.

IX. Definition. Measure, \mathbf{S} , of the distribution of Normal Logarithms is defined for equivalent infinite products \mathbf{A} and \mathbf{B} such that, if,

$$\mathbf{P}_n = \log_A(a_n)$$

or,

$$\mathbf{P}_n = \log_B(b_n)$$

then by Shannon,

$$\mathbf{S} = - \sum_{n=1}^{\infty} \mathbf{P}_n \log(\mathbf{P}_n)$$

Remark. \mathbf{S} is referred to as the entropy of the distribution (Everett).

Example. Consider $\mathbf{P}_n = 1/2^n$,

$$\begin{aligned} \mathbf{S} &= - \sum_{n=1}^{\infty} \frac{1}{2^n} \log\left(\frac{1}{2^n}\right) \\ &= 2 \log(2) \end{aligned}$$

And if the base 2 logarithm is used, $\mathbf{S} = 2 \text{ bits}$

Calculation of $S(\mathbf{x})$ for $0 \leq x < 1$.

$$\frac{1}{1-x} = (1+x) \prod_{n=1}^{\infty} \left(\frac{1+x^{2^n}}{1-x^{2^n}} \right)^{2^{-n}} = B$$

$$\log \frac{1}{1-x} = \log(1+x) + \sum_{n=1}^{\infty} \frac{1}{2^n} \log \left(\frac{1+x^{2^n}}{1-x^{2^n}} \right)$$

$$1 = \log_B(1+x) + \sum_{n=1}^{\infty} \frac{1}{2^n} \log_B \left(\frac{1+x^{2^n}}{1-x^{2^n}} \right) = \sum_{n=1}^{\infty} P_n$$

$$S(\mathbf{x}) = - \sum_{n=1}^{\infty} P_n \log_2(P_n) \text{ (bits)}$$

Figure 2 is a comparison of the entropy of objective state [Model IV] and the proposed entropy of subjective state [Model V]. The area between the curves is the proposed entropy of the objective distortion.

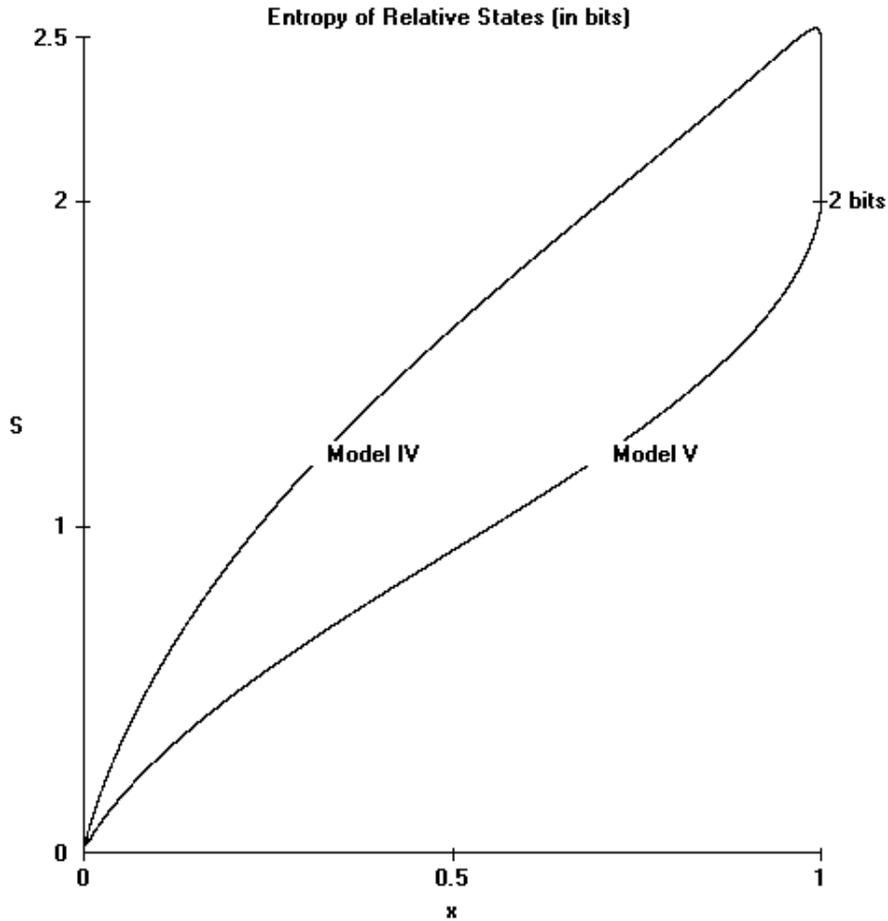


Figure 2. Comparison of State Entropy

5 The Concept of the Entropy of Relative States

The following identity is of utmost importance in any discussion of the information theory of Shannon,

$$Entropy = - \sum_{n=1}^{states} p_n \ln(p_n) = K \ln(states) ,$$

but only for uniform distributions of p_n does the second equality hold. But suppose that the p_n 's are not equal, how do we then compare the entropy of the uniform distribution

against the entropy for the non-uniform distribution? One way is to assume that all finite uniform distributions have entropy of unity and that non-uniform distributions have entropy of less than unity. That is, the non-uniform cases possess more information than the uniform cases. In such a context, we must define a more consistent form of entropy that not only allows for fair comparisons between problems with equal numbers of states, but also allows for fair comparisons between problems possessing different numbers of states. Define Relative Entropy:

$$\mathbf{Entropy} = \frac{Entropy}{\ln(states)} = - \sum_{n=1}^{states} p_n \log_{states}(p_n) .$$

Thus defined, the Relative Entropy may also be interpreted as the Entropy of Relative States. The following dice example should clarify the concept.

Two die each represent a Relative Entropy of unity. That is:

$$\mathbf{Entropy}_{die} = \log_6(6) = 1 .$$

If the 36 possible states of a toss were uniformly distributed, then the Relative Entropy of all possible results would be:

$$\mathbf{Entropy}_{uniform} = \log_{36}(36) = 1 .$$

But the distribution of results is not uniform, so $\mathbf{Entropy}_{toss} =$

$$\begin{aligned} &= \frac{-\left(\frac{2}{36}\right)\ln\left(\frac{1}{36}\right) - \left(\frac{4}{36}\right)\ln\left(\frac{2}{36}\right) - \left(\frac{6}{36}\right)\ln\left(\frac{3}{36}\right) - \left(\frac{8}{36}\right)\ln\left(\frac{4}{36}\right) - \left(\frac{10}{36}\right)\ln\left(\frac{5}{36}\right) - \left(\frac{6}{36}\right)\ln\left(\frac{6}{36}\right)}{\ln(36)} \\ &= 0.6334... \end{aligned}$$

Note that the Notion of Relative Entropy is merely a more precise way to say that $Entropy = K\ln(states)$. The distinguishing property of Relative Entropy is the manner by which it is conserved; e.g., two six state systems, of $Entropy = 1$ each, might be expected to combine into a uniform distribution of twelve states, of $Entropy = 1$, because no information has been added to produce a non-uniform distribution. Due to the rules of dice, thirty-six states result, and likewise we might expect that the thirty-six states

be uniformly distributed and therefore have $Entropy = 1$. The combinations are actually 12345654321, as above, and the dice and rules, amount to -0.3666 units of Relative Information being added to the expected value of $Entropy = 1$ for a uniform thirty-six state system.

Conjecture.

Information is conserved when previously independent systems combine by some set of rules to produce a composite system with a new number of total states. That is, what information was necessary to produce the combination system, can only be determined from the distribution of states, not from the new number of total states.

6 The Concept of Relative Entropy for Finite Products

The concept of entropy in relation to products starts with the need to interpret the distribution of natural logarithms as probabilities:

$$\ln(AB) = \ln(A) + \ln(B)$$

or,

$$\frac{\ln(A)}{\ln(AB)} + \frac{\ln(B)}{\ln(AB)} = P_A + P_B = 1$$

or,

$$\log_{AB}(AB) = \log_{AB}(A) + \log_{AB}(B) = 1$$

P_A and P_B are defined as being intermediate probabilities encountered on the way to the determination of the relative entropy. If A and B are equal, $P_A = P_B = 0.5$, and we are done with the determination of relative entropy. On the other hand, if P_A and P_B are not equal, we are not done and (base $N = 2$ probability states) must be introduced. The additional computations are:

$$Relative\ Entropy = -P_A \log_2(P_A) - P_B \log_2(P_B)$$

$$= \frac{-\frac{\ln(A)}{\ln(AB)} \ln\left(\frac{\ln(A)}{\ln(AB)}\right) - \frac{\ln(B)}{\ln(AB)} \ln\left(\frac{\ln(B)}{\ln(AB)}\right)}{\ln(2)}$$

and when $A = B$,

$$\text{Maximum Relative Entropy} = -\log_2\left(\frac{1}{2}\right) = 1$$

or, in general,

$$\text{Maximum Relative Entropy} = -\log_N\left(\frac{1}{N}\right) = 1$$

for a finite product of N identical factors.

Extension of this concept to infinite products must accompany a constraint that the distribution of factors can never be uniform, but is limited by the convergence properties of the infinite product. That is, a finite value for N can be determined that will limit the Maximum Relative Entropy of Infinite Products to a value of 1 for the “most uniform” distribution of factors encountered over the domain of interest.

7 The Concept of Relative Entropy for Convergent Infinite Products

Combinations or interactions of infinite products do not involve a change in the number of states because the number of states is infinity. This must not be taken as a reason for the entropy to be infinity as it appears to be in the general case; in particular cases where the factors converge to unity over the domain of interest, a uniform distribution of factors is never achieved. Therefore, in this convergent-infinite case, entropy is conserved with respect to any base logarithm that is appropriate. The base two logarithm is chosen because the entropy of the infinite products of this investigation converge to a value of two (bits) as the maximum of their domain is approached ($x \rightarrow 1$). The “unit” of the unitless entropy in (bits) merely signifies that the base of the chosen logarithm is two and therefore must only be compared to entropies computed with the base two logarithm.

8 The Concept of Correlation

The reader must now make a conceptual leap from the notion of equivalence of infinite products to the notion of correlation. Equivalent mathematical structures correlate in the sense that, geometrically, they are like an object container and an object that exactly fits inside the container. In the present discussion the object container is the gravitational field surrounding a central mass (M) and the object contained in it is the collection of all possible circular orbits. The entropies (S), of the objects, are said to be equal when the fit is exact for a particular radius (R) and orbital velocity (v). In other words, the

objects are said to be in exact correlation when their gravitational and inertial entropies are equal ($S_G = S_v$). The “degree of correlation” is less for elliptical orbits trying to fit inside circular gravitational fields, and what correlates (equates) for elliptical orbits are the average entropies. That is,

$$\frac{S_{Gmax} + S_{Gmin}}{2} = \frac{S_{vmax} + S_{vmin}}{2} = \bar{S}_G = \bar{S}_v$$

9 Orbital Interpretation

Infinite products **A** and **B**,

$$A = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = \sqrt{\left(1 + \frac{v^2}{c^2}\right)} \prod_{n=1}^{\infty} \left(\frac{1 + \left(\frac{v^2}{c^2}\right)^{2^n}}{1 - \left(\frac{v^2}{c^2}\right)^{2^n}}\right)^{2^{-(n+1)}},$$

$$B = \frac{1}{1 - \frac{GM}{Rc^2}} = \left(1 + \frac{GM}{Rc^2}\right) \prod_{n=1}^{\infty} \left(\frac{1 + \left(\frac{GM}{Rc^2}\right)^{2^n}}{1 - \left(\frac{GM}{Rc^2}\right)^{2^n}}\right)^{2^{-n}},$$

are equivalent when, $v^2/c^2 = GM/R/c^2$, although their values are related by $A = \sqrt{B}$. This is a case of equivalence, defined previously, when $\rho = 0.5$. That is, when $v = \sqrt{GM/R}$, entropies, \mathbf{S}_A and \mathbf{S}_B are equal. This means that the inertial and gravitational entropies are equal for the circular orbit,

$$v = \sqrt{\frac{GM}{R}},$$

where v is the orbital velocity, G is the gravitational constant, M is the central mass, and R is the distance from M .

The statement that this theory makes concerning General Motion, is based on the principle of the Equivalence of Average Inertial and Average Gravitational Entropies such that,

$$\bar{\mathbf{S}}_{\left(\sqrt{\frac{1}{1 - \frac{v^2}{c^2}}}\right)} = \bar{\mathbf{S}}_{\left(\frac{1}{1 - \frac{GM}{Rc^2}}\right)},$$

from which derives the distortions of length, time, and mass due to motion near a gravitating central mass. Note that the inertial (Lorentz) and gravitational factors are purely themselves (zero correlation), when M or v are respectively zero or when $R = \infty$. At this point, rigorous abstract analysis has failed to predict elliptical motion because of the

lack of theorems to determine the form of the inverse function \mathbf{S}^{-1} of our mathematical object, but its linear approximation shows why elliptical orbital motion is calculated in close agreement with the classical theory of the solar system. In addition, numerical evaluation of \mathbf{S} has shown that: 1) \mathbf{S} vs x has a derivative of zero at $\mathbf{S}_{max} = 2.5307$ (*bits*), $x = 0.99181$, and 2) orbital calculations cannot be made at or above $\mathbf{S} = 2$ (*bits*), $x = 0.7035$ to $x = 1$, because the inverse of \mathbf{S} cannot be defined above $\mathbf{S}_{(2)}^{-1} = 0.7035$.

The generating function for calculation of the entropic ellipse of the computer simulation is:

$$\mathbf{S}_{(x)} = \left(\frac{\mathbf{S}_{max} + \mathbf{S}_{min}}{2}\right) + \left(\frac{\mathbf{S}_{max} - \mathbf{S}_{min}}{2}\right)\cos(\theta)$$

The generated polar coordinate pair, $(\mathbf{S}_{(x)}, \theta)$, is then transformed into (x, θ) by an algorithm that converges on the value for x that corresponds to the desired value of $\mathbf{S}_{(x)}$ generated above. The function will generate a conic ellipse only if $\mathbf{S}_{(x)}$ is linear between x_{min} and x_{max} ; an observation that is consistent with the plot of \mathbf{S} vs x in Figure 1. (for Model IV), but only over portions of the curve that can be assumed linear. This fact can be shown by first converting the polar equation for the *conic ellipse*,

$$\frac{1}{R} = \left(\frac{1}{p}\right) + \left(\frac{e}{p}\right)\cos(\theta),$$

into a representation in terms of R_{min} (perihelion distance) and R_{max} (aphelion distance) instead of a and b , the semi-major and the semi-minor axes. That is, replacing the semi-latus rectum (p) and the eccentricity (e) by their equivalent expressions gives,

$$\frac{1}{R} = \frac{(R_{max} + R_{min})}{(2R_{max}R_{min})} + \frac{(R_{max} - R_{min})}{(R_{max} + R_{min})} \frac{(R_{max} + R_{min})}{(2R_{max}R_{min})}\cos(\theta)$$

And for the *entropic ellipse*,

$$\mathbf{S}_{\left(\frac{1}{R}\right)} = \frac{\mathbf{S}_{\left(\frac{1}{R_{min}}\right)} + \mathbf{S}_{\left(\frac{1}{R_{max}}\right)}}{2} + \frac{\mathbf{S}_{\left(\frac{1}{R_{min}}\right)} - \mathbf{S}_{\left(\frac{1}{R_{max}}\right)}}{2}\cos(\theta)$$

so that, if \mathbf{S} is linear over the sub-domain R_{min} to R_{max} ,

$$\frac{1}{R} = \frac{\left(\frac{1}{R_{min}}\right) + \left(\frac{1}{R_{max}}\right)}{2} + \frac{\left(\frac{1}{R_{min}}\right) - \left(\frac{1}{R_{max}}\right)}{2}\cos(\theta)$$

or,

$$\frac{1}{R} = \frac{(R_{max} + R_{min})}{(2R_{max}R_{min})} + \frac{(R_{max} - R_{min})}{(2R_{max}R_{min})}\cos(\theta),$$

which is exactly the same as for the *conic ellipse*.

To fill the void for theorems concerning the properties of infinite products, an interactive program (orbsim98) has been prepared and can be downloaded from,

http : //EshlemanW.tripod.com/

10 Probability Interpretation

The concept of probability, established earlier, was presented as if it is merely an intermediate step in the determination of the relative entropy of a product representation of the value of a number. I would prefer to interpret the probabilities as observable; showing their presence in a manner that the overall effect obscures their rigorously deterministic values. In general, the probabilities are:

$$P = 1 = \sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} \log_A(a_n) = \sum_{n=1}^{\infty} \frac{\ln(a_n)}{\ln(A)} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\ln(a_n)}{\ln(a_m)}$$

Or, each probability may be said to be the ratio of the logarithm of a part to the logarithm of the whole (a change of base). These probabilities, although deterministic, are indistinguishable upon observation. That the gravitational and inertial probabilities are indistinguishable, is this theory's uncertainty principle governing how much information can be obtained about positions and velocities of interactions involving energies from zero to infinity.

The general properties do not put upper limits on either the value or the entropy of an infinite product, but it is a property of the particular infinite product of this theory that although it is limitless in value (energy), the probabilities converge to a unique non-uniform distribution (entropy) as the energy heads toward infinity. This order out of chaos property, while un-noticed at low inertial and gravitational energies where first pair approximations for the infinite products are accurate and Newtonian, has a profound effect on the structure of this theory's degenerate object, the black-hole of deep-space astronomy.

11 Interpretation of the Degenerate Form

The degenerate form of,

$$\frac{1}{1-x} = (1+x) \prod_{n=1}^{\infty} \left(\frac{1+x^{2^n}}{1-x^{2^n}} \right)^{2^{-n}},$$

as x approaches 1 is,

$$\frac{1}{1-x} = \prod_{n=1}^{\infty} \left(\frac{1}{1-x}\right)^{2^{-n}},$$

with probabilities,

$$\sum_{n=1}^{\infty} P_n = 1 = \sum_{n=1}^{\infty} \frac{1}{2^n},$$

and entropy (using the base 2 logarithm),

$$S_{(1)} = - \sum_{n=1}^{\infty} \frac{1}{2^n} \log_2 \left(\frac{1}{2^n}\right) = \sum_{n=1}^{\infty} \frac{n}{2^n} = 2 \text{ (bits)}.$$

Therefore, the degenerate form of this theory is a determinate fixed structure possessing order. Now, looking at the plot of \mathbf{S} vs x , we see that in addition to (point 3) at $\mathbf{S}_1 = 2$ determined above, there are two other points of special interest on the curve; (point 1) at $\mathbf{S}_{0.7035} = 2$, and (point 2) at $\mathbf{S}_{0.99181} = 2.5307$. That is, the inverse function, \mathbf{S}^{-1} , does not define at or above $x = 0.7035$ and there is a maximum of 2.5307 (and a zero derivative) at $x = 0.99181$. Interpreting x as equal to $GM/R/c^2$, and M as the mass of our Sun, we get a sphere of radius $R = 1,476$ meters enclosing all of M , the one-sol black-hole of this theory. Just above the surface of this black-hole there is a 12 meter thick “boundary-layer” inside which dS/dx is negative and above the “boundary-layer” there is a 610 meter thick layer that fills the majority of the 622 meters where the inverse of its entropy function, \mathbf{S}^{-1} , cannot be defined.

The region where \mathbf{S}^{-1} cannot be defined is interpreted as a region where orbital motion cannot exist. This reasoning is based on the observation that since the numerical simulation generates the entropy polar coordinate pairs, (\mathbf{S}_x, θ) , and needs (but is only numerically capable), \mathbf{S}^{-1} to get the orbital polar pairs, (x, θ) , then analytical methods would likely need \mathbf{S}^{-1} to determine orbits as well. That is, the analytical problem concerning the lack of a defined inverse for a function over some subdomain is mirrored by its numerical simulation.

Of course, this is the structure of perception for a black-hole, distorted by the nature of observation; it is the objective view of the black-hole. Subjectively (independent of observation), neither the event-horizon (as an object) nor the maximum speed of light (as a limit) need apply when the classical notion of subjective state (e^x) is the model. When this theory’s Model 5 is used for subjective state, matter is allowed to attain the maximum speed of light, but is denied faster than light (FTL) motion due to the lack of convergence of Model 5 for values of $x > 1$.

12 Remarks Concerning Quantum Mechanics

The notion of evolution for classical quantum mechanics is,

$$\left[\frac{f_1}{f_0}\right]^t = [e^H]^t = e^{tH} ,$$

where H is a constant Hamiltonian operator that is independent of time (t). Furthermore, interpretation of H as a sum gives,

$$\left[\frac{f_1}{f_0}\right]^t = [e^{\sum_n H_n}]^t = e^{t \sum_n H_n} .$$

Now, assuming a product form, $e^H = \prod_n h_n$ for the relative state,

$$\left[\frac{f_1}{f_0}\right]^t = \left[\prod_n h_n\right]^t = \left[\prod_n e^{\ln(h_n)}\right]^t = [e^{\sum_n \ln(h_n)}]^t = e^{t \sum_n \ln(h_n)} ,$$

an unremarkable result in that it merely demonstrates a longer way to the same answers if $h_n = e^{H_n}$; or so it would seem. What the above relation does show is that if a candidate for a product version of the relative state is to be tested, then $H_n = \ln(h_n)$ must agree with experimental results.

For example, the special relativistic mass increase with velocity requires that the relative state must be,

$$\frac{f_1}{f_0} = \sqrt{\frac{1}{1 - v^2/c^2}} = \sqrt{1 + v^2/c^2} \prod_{n=1}^{\infty} \left[\frac{1 + (v^2/c^2)^{2^n}}{1 - (v^2/c^2)^{2^n}} \right]^{2^{-(n+1)}} = e^{H_v} ,$$

where the special relativistic Hamiltonian is,

$$H_v = \frac{1}{2} \ln(1 + v^2/c^2) + \sum_{n=1}^{\infty} \left[\frac{1}{2^{n+1}} \ln(1 + (v^2/c^2)^{2^n}) \right] - \sum_{n=1}^{\infty} \left[\frac{1}{2^{n+1}} \ln(1 - (v^2/c^2)^{2^n}) \right] ,$$

and likewise for gravity,

$$H_g = \ln(1 + GM/R/c^2) + \sum_{n=1}^{\infty} \left[\frac{1}{2^n} \ln(1 + (GM/R/c^2)^{2^n}) \right] - \sum_{n=1}^{\infty} \left[\frac{1}{2^n} \ln(1 - (GM/R/c^2)^{2^n}) \right] .$$

H_v and H_g are energy operators of the mass m , so the evolution of state is now,

$$e^{tHm} = e^{t(H_v - H_g)m} .$$

One might think that the above Hamiltonian, $H = H_v - H_g$, is insufficient in that the momentum displacement seems to be missing, but the reasoning here is that momentum displacement is fictitious. That is, instead of a priori assuming momentum to place the planets in their proper orbits, we here assume that stationary states (closed orbits) are due to the equivalence (correlation) of the average distributions of the inertial and gravitational Hamiltonians above. Therefore, there objectively appears to be a real property of matter called momentum, but it is fictitious in that it is merely the result of a correlation between motion and gravity that actually causes the “momentum” displacement. The fiction is that momentum is a cause; the suggested truth is that momentum is an effect. If one is so bold as to fictionalize momentum, then, to be consistent, magnetism must also be fictionalized. That is, the orbit of an electron is a relationship between Hamiltonian distributions; the magnetism is but a result of this relationship, not a partial cause of the relationship.

According to the method of this report we suggest that Hamiltonians containing momentum and/or magnetism terms are overdetermined because sufficient information for prediction of these phenomena are contained in the distributions of motion and gravitational/coulomb displacements. Furthermore, it is suggested that it is the fictionalized displacements that are subjectively quantized, whereas what the fundamental causes describe is an objectively continuous coordinate change. In other words, the proposed Hamiltonians for the motion and for the attraction, together with the correlation machinery described above, form a sufficient and logically self-consistent mathematical description of the evolution of states in a universe in which many observers are at work.

Since relativistic inertial and gravitational Hamiltonians have been determined, a similar approach to quantum mechanics could lead to a unified formulation. This method of factorization of the relative state is not exhausted by the infinite products presented in this report as there are at least as many infinite product identities as there are number system bases.

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